SRI CHAITANYA EDUCATIONAL INSTITUTIONS,A.P.

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ENGINEERING *EAMCET-2012* *DATE: 12-05-12*

SUB: *MATHS* *MARKS: 80*

1. The equation of a straight line passing through the point (1, 2) and inclined at 45^0 to the line y = 2x + 1 is

1) 5x + y = 7 2) 3x + y = 5 3) x + y = 3 4) x – y + 1 = 0

SOL: Slope$=\dfrac{m+\tan\alpha}{1-m\tan\alpha}\, or\, \dfrac{m-\tan\alpha}{1+m\tan\alpha}$

$= -3, \dfrac{1}{3}$ equation of line 3x + y = 5

2. A point moves in the xy – plane such that the sum of its distance from two mutually perpendicular lines is always equal to 5 units. The area (in square units) enclosed by the locus of the point, is

1) $\dfrac{25}{4}$ 2) 25 3) 50 4) 100

SOL: $|x|+|y|=5$ area = $2c^2$ = 50

3. The distance between the parallel lines given by $(x + 7y)^2 +$ $4\sqrt{2}\left(x+7y\right)-42=0$ **is**

1) $\dfrac{4}{5}$ 2) $4\sqrt{2}$ 3) 2 4) $10\sqrt{2}$

SOL: Put x + 7y = t $t^2+4\sqrt{2}t-42=0$

$t=\dfrac{-4\sqrt{2}\pm\sqrt{32+168}}{2} = \dfrac{-4\sqrt{2}\pm10\sqrt{2}}{2} = 3\sqrt{2}, -7\sqrt{2}$

$x+7y-3\sqrt{2}=0, x+7y+7\sqrt{2}=0$ Distance $=\dfrac{2\cancel{10}\,\cancel{\sqrt{2}}}{\cancel{5}\,\cancel{\sqrt{2}}}=2$

4. If the area of the triangle formed by the pair of lines $8x^2 – 6xy + y^2 = 0$ and the line 2x + 3y = a is 7 then a =

1) 14 2) $14\sqrt{2}$ 3) $28\sqrt{2}$ 4) 28

SOL: Given area = 7

$\Rightarrow \dfrac{n^2\sqrt{h^2-ab}}{\left|am^2-2hlm+bl^2\right|}=7$

$\Rightarrow \dfrac{a^2\sqrt{9-8}}{\left|72+36+4\right|}=7$ $\Rightarrow a^2=7x112=784 \Rightarrow$ a = 28

5. If the pair of lines given by $\left(x^2+y^2\right)\cos^2\theta=\left(x\cos\theta+y\sin\theta\right)^2$ are perpendicular to each other, then θ =

1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $3\frac{\pi}{4}$

SOL; Coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \cos^2\theta=\sin^2\theta \Rightarrow \quad \tan^2\theta=1\Rightarrow \tan\theta=\pm1\Rightarrow \theta=\frac{\pi}{4}(or)\frac{3\pi}{4}$$

6. Given the circle C with the equation $x^2 + y^2 - 2x + 10y - 38 = 0$.

Match the List – I with the List – II given below concerning C:

List – I	**List – II**
i) The equation of the polar of (4,3) with respect to C	**a) y + 5 = 0**
ii) The equation of the tangent at (9, –5) on C	**b) x = 1**
iii) The equation of the normal at (–7, –5) on C	**c) 3x + 8y = 27**
iv) The equation of the diameter of C passing through (1, 3)	**d) x + y = 3**
	e) x = 9

The correct answer is

	i	ii	iii	iv		i	ii	iii	iv
1)	c	a	e	b	2)	d	e	a	b
3)	c	e	a	b	4)	d	b	a	e

SOL: i) polar equation is $s_1=0\rightarrow 3x+8y-27=0$

ii) tangent at (9, –5) is $s_1=0\rightarrow x-9=0$

iii) equation of normal is passing through (–7, –5) and $\rightarrow$ $y+5=0$

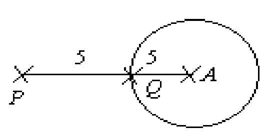
centre (1, –5)

iv) equation of diametre is $\rightarrow$ x – 1 = 0

7. Consider the circle $x^2 + y^2 - 4x - 2y + c = 0$ whose centre is A(2, 1). If the point P(10, 7) is such that the line segment PA meets the circle in Q with PQ = 5, then c =

1) – 15 2) 20 3) 30 4) – 20

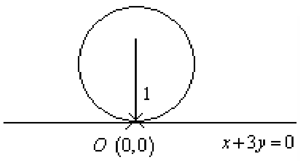
SOL: AP = 10



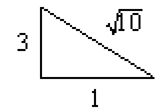
Q = Mid point of AP = (6, 4) (6, 4) lies on O[le] $\therefore c=-20$

8. If the line x + 3y = 0 is the tangent at (0, 0) to circle of radius 1, then the centre of one such circle is

1) (3,0) 2) $\left(\frac{-1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$ 3) $\left(\frac{3}{\sqrt{10}}, \frac{-3}{\sqrt{10}}\right)$ 4) $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$

SOL: 

Slope $= \frac{-1}{3}$, $\perp slope = \tan\theta = 3$



centre = $\left(0 \pm 1\left(\frac{1}{\sqrt{10}}\right), 0 \pm 1\left(\frac{3}{\sqrt{10}}\right)\right) = \left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$

9. A circle passes through the point (3,4) and cuts the circle $x^2 + y^2 = a^2$ orthogonally; the locus of its centre is a straight line. If the distance of this straight line from the origin is 25, then a^2 =

1) 250 2) 225 3) 100 4) 25

SOL: $(x-3)^2 + (y-4)^2 = 0 \ \rightarrow (1)$

$x^2 + y^2 - a^2 = 0 \rightarrow (2)$

R.A is $6x - 8y - 25 - a^2 = 0 \rightarrow (3)$

given distance from $(0,0)$ **to (3) =** a^2

$$\frac{\left|-25 - a^2\right|}{\sqrt{36+64}} = 25 \rightarrow \left|25 + a^2\right| = 250$$

$a^2 = 225$

10. The equation to the line joining the centres of the circles belonging to the coaxial system of circles $4x^2 + 4y^2 - 12x + 6y - 3 + \lambda(x + 2y - 6) = 0$ **is**

1) 8x - 4y - 15 = 0 2) 8x - 4y + 15 = 0 3) 3x - 4y - 5 = 0 4) 3x - 4y + 5 = 0

SOL: R.A axis is x + 2y-6 =0, lines of centers is $2x - y + k = 0$

clearly centre $\left(\frac{3}{2}, -\frac{3}{4}\right)$ lies on it $\Rightarrow$ $2\left(\frac{3}{2}\right) + \frac{3}{4} + k = 0 \Rightarrow$

$\frac{12+3}{4} + k = 0 \Rightarrow k = \frac{-15}{4}$

$$\therefore 2x - y \frac{-15}{4} = 0 \Rightarrow 8x - 4y - 15 = 0$$

11. Let x + y = k be a normal to the parabola $y^2 = 12x$. If p is length of the perpendicular from the focus of the parabola onto this normal, then $4k - 2p^2 = 0$

1)1 2)0 3) -1 4)2

SOL: Compare with normal of $y^2 = 12x$ we get K=9

S = (3,0)

also $p = \frac{|3(1)-9|}{\sqrt{2}} = \frac{6}{\sqrt{2}}$

$\therefore 4k - 2p^2 = 36 - 2\left(\frac{36}{2}\right) = 0$

12. If the line 2x + 5y = 12 intersects the ellipse $4x^2 + 5y^2 = 20$ in two distinct point A and B, then mid point of AB is

1) (0,1) 2) (1,2) 3) (1,0) 4) (2,1)

SOL: Clearly by verfication only option (2) lies on the given line

13. Equation of one of the tangents passing through (2,8) to the hyperbola $5x^2 - y^2 = 5$ is

1) 3x + y - 14 = 0 2) 3x - y + 2 = 0 3) x + y + 3 = 0 4) x - y + 6 = 0

SOL: Hyperbola is $\frac{x^2}{1} - \frac{y^2}{5} = 1$, equation of tangent is $y = mx \pm \sqrt{m^2 - 5}$

but (2,8) lies on it $\Rightarrow (8-2m)^2 = (m^2 - 5) \Rightarrow m = 3$ $\therefore 3x - y + 2 = 0$

14. The area (in square units) of the equilateral triangle formed by the tangent at $(\sqrt{3}, 0)$ to the hyperbola $x^2 - 3y^2 = 3$ with the pair of asymptotes of the hyperbola is

1) $\sqrt{2}$ 2) $\sqrt{3}$ 3) $\frac{1}{\sqrt{3}}$ 4) $2\sqrt{3}$

SOL: Tangent **is** $s_1 = 0$ **is** $x(\sqrt{3}) = 3 \rightarrow x = \sqrt{3} \rightarrow (1)$

Asymptotis are $x + \sqrt{3}y = 0 \rightarrow (2)$ $x - \sqrt{3}y = 0 \rightarrow (3)$

P .I are $(0,0), (\sqrt{3}, -1); (\sqrt{3} : 1)$ $\therefore$ **area =** $\frac{1}{2}|2\sqrt{3}| = \sqrt{3}$

15. The radius of the circle r = 12 $\cos\theta + 5\sin\theta$ is

1) $\frac{5}{12}$ 2) $\frac{17}{2}$ 3) $\frac{15}{2}$ 4) $\frac{13}{2}$

Sol: $r = 12\frac{x}{r} + \frac{5y}{r}$

$r^2 = 12x + 5y$

$x^2 + y^2 - 12x - 5y = 0$ $r = \sqrt{36 + \frac{25}{4}} = \sqrt{\frac{144+25}{4}} = \sqrt{\frac{169}{4}} = \frac{13}{2}$

16. If x - coordinate of a point P on the line joining the points Q(2,2,1) and R(5,1,-2) is 4, then the z -coordinate of P is

1) -2 2) -1 3) 1 4) 2

Sol: P divides QR in the ratio = $x_1 - x : x - x_2$

$$= 2 - 4 : 4 - 5$$

$$= -2 : -1 = 2 : 1$$

$$P = \left(\frac{10+2}{3}, \frac{2+2}{3}, \frac{-4+1}{3} \right)$$

$$= \left(4, \frac{4}{3}, -1 \right)$$

17. A straight line is equally inclined to all the three coordinate axes. Then an angle made by the line with the y-axis is

1) $\cos^{-1}\left(\frac{1}{3}\right)$ 2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 3) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ 4) $\frac{\pi}{4}$

SOL: put l = m = n in $l^2 + m^2 + n^2 = 1 \Rightarrow 3l^2 = 1 \Rightarrow l = \pm \frac{1}{\sqrt{3}}$

$\therefore$ angle made by line in with y^- axis $= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

18. If the foot of the perpendicular from (0,0,0) to a plane is (1,2,3), then the equation of the plane is

1) 2x + y + 3z = 14 2) x + 2y + 3z = 14 3) x + 2y + 3z + 14 = 0 4) x + 2y - 3z = 14

SOL: Let P = (x, y, z)

equation of $\overline{QR}$ is $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$

put (x, y,z) on it $x^2 + y^2 - 12x - 5y = 0$ $r = \sqrt{36 + \frac{25}{4}} = \frac{13}{2}$

19. The equation of the sphere through the points (1,0,0) (0,1,0) and (1,1,1) and having the smallest radius

1) $3(x^2 + y^2 + z^2) - 4x - 4y - 2z + 1 = 0$ **2) $2(x^2 + y^2 + z^2) - 3x - 3y - z + 1 = 0$**

3) $x^2 + y^2 + z^2 - x - y + z + 1 = 0$ **4)** $x^2 + y^2 + z^2 - 2x - 2y + 4z + 1 = 0$

Sol: Triangle is an equalateral triangle. Centre is centroid and radius is circum radius.

20. $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4} =$

1) e^4 2) e^6 3) e^5 4) e

Sol: $= e^{6-1} = e^5$

21. Let $f: R \to R$ **be defined by** $f(x) = \begin{cases} \alpha + \frac{\sin[x]}{x} & \text{if } x > 0 \\ 2 & \text{if } x = 0 \\ \beta + \left[\frac{\sin x - x}{x^3}\right] & \text{if } x < 0 \end{cases}$

where [y] denotes the integral part of y. If f continous at x = 0, then $\beta - \alpha =$

1) -1 2) 1 3) 0 4) 2

Sol: $\alpha + 1 = 2$

$\alpha = -2$

$\alpha + 1 = B$

$1 = \beta - \alpha$

22. $f(x) = \log\left(e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right) \Rightarrow f^1(0)$

1) $\frac{1}{4}$ 2) 4 3) $\frac{-3}{4}$ 4) 1

SOL; $f(x) = x + \frac{3}{4}\left[\log(x-1) - \log(x+2)\right]$

$f^{|}(x) = 1 + \frac{3}{4}\left[\frac{1}{x-2} - \frac{1}{x+2}\right] = 1 + \frac{3}{x^2-4}$ $\Rightarrow f^{|}(0) = 1 - \frac{3}{4} = \frac{1}{4}$

23. If $xy \neq 0, x + y \neq$ **0 and** $x^m y^n = (x+y)^{m+n}$ **where** $m, n \notin N$ *then* $\frac{dy}{dx} =$

1) $\frac{y}{x}$ 2) $\frac{x+y}{xy}$ 3) xy 4) $\frac{x}{y}$

SOL; homegeneous function of degree of $m+n$ $\frac{dy}{dx} = \frac{y}{x}$

24. $x^2 + y^2 = t + \frac{1}{t}, x^4 + y^4 = t^2 + \frac{1}{t^2} \Rightarrow x^3 y \frac{dy}{dx} =$

1) -1 2) 1 3) 0 4) t

SOL: $x^4 + y^4 = t^2 + \frac{1}{t^2}$ $= \left(t^2 + \frac{1}{t^2}\right)^2 - 2$ $= \left(x^2 + y^2\right)^2 - 2$ $= x^4 + y^4 + 2x^2y^2 - 2$

$x^2y^2 = 1$ $y^2 = \frac{1}{x^2}$ $2y\frac{dy}{dx} = \frac{-2}{X^3}$ $x^3 y \frac{dy}{dx} = -1$

25. $f(x) = (x^2 - 1)^7 \Rightarrow f^{(14)}(x) =$

1) 0 2) 2! 3) 7! 4) 14!

Sol: Degree of f(x) is 14 & Coefficient of x^{14} **is 1.** $\Rightarrow f^{(14)}(x) = 14!$

26. **The coordinates of the point P on the curve** $x = a\left(\theta + \sin\theta\right), y = a\left(1 - \cos\theta\right)$ **where the tangent is iclined at an angle** $\frac{\pi}{4}$ **to x-axis, are**

1) $\left(a\left(\frac{\pi}{4} - 1\right), a\right)$ 2) $\left(a\left(\frac{\pi}{2} + 1\right), a\right)$ 3) $\left(a\frac{\pi}{2}, a\right)$ 4) $\left(a, a\right)$

SOL; $\frac{dy}{dx} = \frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$ $\tan\frac{\theta}{2} = 1 \Rightarrow \theta = \frac{\pi}{2}$ $\left(a\left(\frac{\pi}{2} + 1\right), a\right)$

27. **If** Δ **is the area of the traingle formed by the positive x-axis and the normal and tangent to the circle** $x^2 + y^2 = 4$ **at** $\left(1, \sqrt{3}\right)$**, then** Δ **=**

1) $\frac{\sqrt{3}}{2}$ 2) $\sqrt{3}$ 3) $2\sqrt{3}$ 4) 6

SOL: $x^2 + y^2 = 4$ Diff w.r.t 'x' $(x_1, y_1) = (1, \sqrt{3})$

$\frac{dy}{dx} = \frac{-x}{y}$, $m = \frac{-1}{\sqrt{3}}$ Area = $\left|\frac{y_1^2(1+m^2)}{2m}\right| = 2\sqrt{3}$

28. If the volume of a sphere increases at the rate of $2\pi\, cm^3/\sec$, then the rate of increase of its radius $(in\, cm/\sec)$, when the volume is $288\pi\ cm^3$ is

1) $\frac{1}{36}$ 2) $\frac{1}{72}$ 3) $\frac{1}{18}$ 4) $\frac{1}{9}$

SOL: $\frac{du}{dt} = 2\pi\, cm^3/\sec$

$V = \frac{4}{3}\pi r^3$

$r^3 = 6^3$

r = 6

$\frac{dV}{dt} = 4\pi r^2 \frac{dv}{dt}$

$\frac{2\pi}{4\pi \times 6 \times 6} = \frac{dv}{dt}$

$\frac{dv}{dt} = \frac{1}{72}\ \ cm/\sec$

29. If $u = f(r)$, where $r^2 = x^2 + y^2$ then $\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) =$

1) $f''(r)$ 2) $f''(r) + f^1(r)$ 3) $f''(r) + \frac{1}{r} f'(r)$ 4) $f''(r) + rf'(r)$

SOL: $u_x = f^1(x)\sqrt{x} = f^1(x)\left(\frac{x}{r}\right)$

$u_{xx} = f^{11}(x)\frac{x^2}{r^2} + f^1(x)\frac{r - x\, x/r}{r^2} = f^{11}(x)\frac{x^2}{r^2} + f^1(x)\frac{r^2 - x^2}{r^3}$

$u_{yy} = f^{11}(r)\frac{y^2}{r^2} + f^1(x)\frac{r^2 - y^2}{r^3}$

$u_{xx} + u_{yy} = f^{11}(r) + f^1(x)\frac{1}{r}$

30. $\int \frac{dx}{x^2\sqrt{4 + x^2}} =$

1) $\frac{1}{4}\sqrt{4 + x^2} + c$ 2) $\frac{-1}{4}\sqrt{4 + x^2} + c$ 3) $\frac{-1}{4x}\sqrt{4 + x^2} + c$ 4) $\frac{9}{4x}\sqrt{4 + x^2} + c$

SOL: $\int \frac{dx}{x^3\sqrt{\frac{4}{x^2} + 1}}$

$\frac{4}{x^2} + 1 = t$

$\frac{-1}{8}\int \frac{dt}{\sqrt{t}}$

$\frac{-8}{x^3}dx = dt$

$= \frac{-1}{4}\sqrt{t} + c = \frac{-1}{4x}\sqrt{4 + x^2} + C$

31. $\int \sec^2 x \cos ec^4 x\, dx = -\frac{1}{3}\cot^3 x + k\tan x - 2\cot x + c \Rightarrow k =$

1) 4 2) 3 3) 2 4) 1

SOL: $\int \frac{dx}{\sin^4 x \cos^2 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^4 x \cos^2 x} dx$

$= \int \frac{dx}{\sin^2 x \cos^2 x} + \int \frac{dy}{\sin^4 x}$

$= \int \left(\sec^2 x + \cos ec^2 x\right) dx + \int \cos ec^4 x dx$

$= \tan x - \cot x + \int \cos ec^2 x\left(1 + \cot^2 x\right)$

$= \tan x - \cot x - \cot x - \frac{\cot^3 x}{3}$

32. $\int \frac{dx}{\sqrt{x - x^2}} =$

1) $2\sin^{-1}\sqrt{x} + c$ 2) $2\sin^{-1} x + c$ 3) $2x\sin^{-1} x + c$ 4) $\sin^{-1}\sqrt{x} + c$

SOL: $\int \frac{dx}{\sqrt{x}\sqrt{1-x}}$ $\sqrt{x} = \sin\theta$ $\frac{1}{2\sqrt{x}} dx = \cos\theta d\theta$

$= \int \frac{2\cos\theta d\theta}{\sqrt{1 - \sin^2\theta}} = 2\theta + c = 2\sin^{-1}\sqrt{x} + c$

33. $a > 0, \int_{-\pi}^{\pi} \frac{\sin^2 x}{1 + a^x} dx =$

1) $\frac{\pi}{2}$ 2) π 3) $\frac{2\pi}{2}$ 4) $a\pi$

SOL: $I = \int_{-\pi}^{\pi} \frac{\sin^2 x}{1 + a^x} dx = \int_{-\pi}^{\pi} \frac{\sin^2(-x)}{1 + a^{-x}} dx = \int_{-\pi}^{\pi} \frac{a^x \sin^2 x}{1 + a^x} dx$

$2I = \int_{-\pi}^{\pi} \sin^2 x\ dx = = 2\int_0^{\pi} \sin^2 x\ dx = = 2\left(2\int_0^{\pi} \sin^2 x\ dx\right) = 4I_2 = 4\frac{1}{2} \times \frac{\pi}{2} = \pi$

34. The area (in square units)bounded by the curves $y^2 = 4x \text{ and } x^2 = 4y$ is

1) $\frac{64}{3}$ 2) $\frac{16}{3}$ 3) $\frac{8}{3}$ 4) $\frac{2}{3}$

SOL: Area bonded by $y^2 = 4ax,\ x^2 = 4ay$ is $\frac{16}{3}a^2$ Sq units

35. The value of the integral $\int_0^4 \frac{dx}{1 + x^2}$ obtained by using Trapezoidal rule with $h = 1$ is

1) $\frac{63}{85}$ 2) $\tan^{-1}(4)$ 3) $\frac{108}{85}$ 4) $\frac{113}{85}$

SOL:

x	0	1	2	3	4
$f(x)=\frac{1}{1+x^2}$	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$

T.R is $\int_a^b f(x)dx = \frac{n}{2}\left[(y_0+y_n)+2(y_1+y_2+.....)\right]$

$\int_0^4 \frac{1}{1+x^2}dx = \frac{1}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right] = \frac{1}{2}\left[\left(1+\frac{1}{17}\right)+2\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}\right)\right] = \frac{113}{85}$ (key - 4)

36. $\frac{dy}{dx}+2x\tan(x-y)=1 \Rightarrow \sin(x-y)=$

1) Ae^{-x^2} 2) Ae^{2x} 3) Ae^{x^2} 4) Ae^{-2x}

SOL: Put x - y = t = $\Rightarrow \frac{dy}{dx}=1-\frac{dt}{dx}$

given equation becomes 2x tant = $\frac{dt}{dx}$

$x^2 = \log(\sin t + \log c)$ $\sin t = Ae^{x^2}$

$\sin(x-y)=Ae^{x^2}$ (key - 3)

37. An integrating factor of the differential equation $\left(1+x^2\right)\frac{dy}{dx}+xy=\frac{x^4}{\left(1+x^5\right)}\left(\sqrt{1-x^2}\right)^3$ **is**

1) $\sqrt{1-x^2}$ 2) $\frac{x}{\sqrt{1-x^2}}$ 3) $\frac{x^2}{\sqrt{1-x^2}}$ 4) $\frac{1}{\sqrt{1-x^2}}$

SOL: $(1-x^2)\frac{dy}{dx}+xy=\frac{x^4}{1+x^5}\sqrt{1-x^2}(1-x^2)$

Divide on both sides with $(1-x^2)$

$\frac{dy}{dx}+\frac{xy}{1-x^2}=\frac{x^4.\sqrt{1-x^2}}{1+x^5}$

I.F = $e^{\int\frac{x}{1-x^2}dx} = e^{\frac{-1}{2}\int\frac{-2x}{1-x^2}} = e^{\frac{-1}{2}\log(1-x^2)} = \frac{1}{\sqrt{1-x^2}}$ (key - 4)

38. If $f: R \to R^2$ and $g: R^+ \to R$ are such that $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then a possible choice for f and b is

1) $f(x) = x^2, g(x) = \sin\sqrt{x}$
2) $f(x) = \sin x, g(x) = |x|$
3) $f(x) = \sin x^2, g(x) = \sqrt{x}$
4) $f(x) = x^2, g(x) = \sqrt{x}$

SOL: $g(f(x)) = \sqrt{\sin^2 x} = |\sin x|$ $f(g(x)) = \left(\sin\sqrt{x}\right)^2$

39. If z → z is $f: z \to z$ **is defined by** $f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ 0 & \text{if } x \text{ is odd} \end{cases}$ **then f is**

1) onto but not one to one
2) one to one but not onto
3) one to one and onto
4) neither one to one nor onto

SOL: f(1) = 0 = f(3)

$x = 0, \pm2, \pm4, \pm6.......\infty$

$\frac{x}{2} = 0, \pm1, \pm2, \pm3.......\infty + 2$

$\therefore$ onto but not 1-1

40. If $\frac{1}{2\times4} + \frac{1}{4\times6} + \frac{1}{6\times8} + ...$ **(n-terms) =** $\frac{kn}{n\times1}$ **then k =**

1) $\frac{1}{4}$
2) $\frac{1}{2}$
3) 1
4) $\frac{1}{8}$

SOL: $\frac{1}{2}\left[\frac{1}{2} - \frac{1}{4} + \frac{1}{4} - \frac{1}{6} + ... + \frac{1}{2n} - \frac{1}{2n+2}\right] = \frac{1}{4}\left[1 - \frac{1}{n+1}\right] = \frac{1}{4}\left[\frac{n}{n+1}\right]$ $\therefore k = \frac{1}{4}$

41. A regular polygon of n sides has 170 diagonals. then n =

1) 12
2) 17
3) 20
4) 25

SOL: No. diagonals $= \frac{n(n-3)}{2} = 170$

n(n-3) = 340 = 20 $\times$ 17

n = 20

42. A committee of 12 members is to be formed from 9 women and 8 men. The number of comittees in which the women are in majority is

1) 2720 2) 2702 3) 2270 4) 2278

Sol: $^9c_9.^8c_3+^9c_8.^8c_4+^9c_7.^8c_5$

$1.\frac{8.7.6}{3.2.1}+9.\frac{8.7.5}{4.3.2.1}+\frac{9.8}{2.1}.\frac{8.7.6}{3.2.1}$

$56+630+(36)(56)=2702$

43. A student has to answer 10 out of 13 questions in an examination choosing at least 5 questions from the first 6 questions. The number of choice available to the student is

1) 63 2) 91 3) 161 4) 196

SOL: $6_{c_5}.7_{c_5}+6_{c_6}.7_{c_4}$

$\frac{6.73}{\cancel{2}.1}+1.\frac{7.\cancel{6}5.\cancel{4}}{\cancel{4}.3.\cancel{2}.1}$

126+35

161

44. $\sum_{k=1}^{\infty}\sum_{r=0}^{k}\frac{1}{3^k}\left({}^kC_r\right)=$

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) 1 4) 2

SOL: $\sum_{k=1}^{r}\left(\frac{1}{3^k}k_{c_0}+k_{c_1}+k_{c_2}----+k_{c_k}\right)$ $\sum_{k=1}^{\infty}\left(\frac{2}{3}\right)^k=\frac{2}{3}+\left(\frac{2}{3}\right)^1-----=\frac{2/3}{1-2/3}=2$

45. If $ab\neq 0$ **and the sum of the coefficients of** x^7 **and** x^4 **in the expansion of** $\left(\frac{x^2}{a}-\frac{b}{x}\right)^{11}$

1) a = b 2) a + b = 0 3) ab = -1 4) ab = 1

SOL: $=\frac{11.2-7}{2+1}+1=T_6$

$=\frac{11.2-4}{2+1}+1=T_7$

$T_6+T_7=11_{C_5}\left(\frac{1}{a}\right)^6(-b)^5+11_{C_6}\left(\frac{1}{a}\right)^5(-b)^6=0$ $ab=1$

46. $\frac{1}{x(x+1)(x+2)....(x+n)} = \frac{A_0}{x} + \frac{A_i}{x+1} + ...\frac{A_n}{x+n}, 0 \le r \le \Rightarrow A_r =$

1) $(-1)^r \frac{r!}{(n-r)!}$ 2) $(-1)^r \frac{1}{r!(n-r)!}$ 3) $\frac{1}{r!(n-r)!}$ 4) $\frac{r!}{(n-r)!}$

SOL: By verification with $r=1, n=2$ and $r=3, n=4$

47. $1 + \frac{1}{3.2^2} + \frac{1}{5.2^4} + \frac{1}{7.2^6} + ... =$

1) $\log_e^2$ 2) $\log_e^3$ 3) $\log_e^4$ 4) $\log_e^5$

SOL: $2\left[\frac{1/2}{1} + \frac{1/2^3}{3} + \frac{1/2^5}{5} - -\right]$

$\log\left(\frac{1+1/2}{1-1/2}\right) = \log\frac{3/2}{1/2} = \log_e 3$

48. $\angle R = \frac{\pi}{4}$**,** $\tan\left(\frac{P}{3}\right), \tan\left(\frac{Q}{3}\right)$ **are the roots of the equation $ax^2 + bx + c = 0$, then**

1) a + b = 0 2) b + c = 0 3) a + c = 0 4) b = c

SOL: $R = \frac{\pi}{4} \Rightarrow p + q = \frac{3\pi}{4}$ $\frac{p}{3} + \frac{q}{3} = \frac{\pi}{3}$

$\frac{\tan\frac{p}{3} + \tan\frac{q}{3}}{1 - \tan\frac{p}{3}\tan\frac{q}{3}} = 1$ $-\frac{b/a}{1-c/a} = 1$ $-b = a - c$ **a+b =c**

49. **The product of real of the equation** $|x|^{\frac{6}{5}} - 26|x|^{\frac{3}{5}} - 27 = 0$

1) -3^{10} 2) -3^{12} 3) $-3^{\frac{12}{5}}$ 4) $-3^{\frac{21}{5}}$

SOL: $|x|^{3/5} = t$

$|x|^{3/5} = t$

$t^2 - 26t - 27 = 0$

$t = 27 or -1$

$|x|^{3/5} = t = 27 only$ **product of x values =** $3^5\left(-3^5\right) = -3^{10}$

50. **If α,β,γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ then the coefficient of x in the cubic equation whose roots are $\alpha(\beta+\gamma),\beta(\gamma+\alpha)$ and $\gamma(\alpha+\beta)$ is**

1) 2q 2) $q^2 + pr$ 3) $p^2 - qr$ 4) r(pq - r)

SOL: α,β,γ **roots of Given** $f(x) = x^3 + px^2 + qx + r = 0$

$$\therefore \alpha+\beta+\gamma = -p$$
$$\alpha\beta+\beta\gamma+\gamma\alpha = q$$
$$\alpha\beta\gamma = -r$$

Let $y = \alpha(\beta+\gamma) = \alpha\beta + \alpha\gamma + \beta\gamma - \frac{\beta\gamma\alpha}{\alpha}$

$$y = q + \frac{r}{\alpha} \quad \therefore \alpha = \frac{r}{y-q}$$

$$\Rightarrow \frac{r^3}{(x-q)^3} + \frac{pr^2}{(x+q)^2} + \frac{qr}{(x-q)} + r = 0$$

$$\Rightarrow (x-q)^3 + q(x-q)^2 + pr(x-q) + r^2 = 0$$

$\therefore$ **coefficient of** $x = q^2 + pr$

51. **Let** $A = \begin{vmatrix} 2 & e^{i\pi} \\ i & i^{2012} \end{vmatrix}$ $C = \frac{d}{dx}\left(\frac{1}{x}\right)_{x=1}$ $D = \int_{e^2}^{1} \frac{dx}{x}$ **. If the sum of two roots of the equation $Ax^3+Bx^2+Cx+D = 0$ is equal to zero, then B =**

1) -1 2) 0 3) 1 4) 2

SOL: **A = 1, B =? , C = -1, D= -2**

Sub in $Ax^3 + bx^2 + cx + D = 0$

$$x^3 + Bx^2 - x + 2 = 0$$
$$\alpha + \beta + \gamma = B$$

r = -B $(\alpha + \beta = 0)$

Sub in given equal

$$B^3 + B^3 + B - 2 = 0$$

B=2

52. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \Rightarrow A^8$

1) 4B 2) 8 B 3) 64B 4) 128 B

SOL: $A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$= \begin{bmatrix} -2 & +2 \\ +2 & -2 \end{bmatrix} = -2B$

$A^8 = \left(A^2\right)^4 = \left(-2B\right)^4 = 16B^4$

$B^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & i \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2B$

= 16.4.2B

= 128 B

53. $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & (x-1)x(x+1) \end{vmatrix} \Rightarrow f(2012) =$

1) 0 2) 1 3) –500 4) 500

SOL: $f(x) = 0$

$f(2012) = 0$

54. Let $A = \begin{bmatrix} -1 & -2 & -3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, if a, b and c respectively denote the ranks the ranks of A, B and C then the correct order of these number is

1) a < b < c 2) c < b < c 3) b < a < c 4) a < c < b

SOL: Range of A = a = 2

B = b = 1

b < c < a

55. Given that $a^2 \quad \neq$ and that the system of equations

$(a \quad)$

$(b \quad)$

$(a \quad) \quad (\alpha \quad)$

has a non - trivial solution, then a, b,c lie in

1) Arithmetic progression 2) Geometric progression

3) harmonic progression 4) Arithmetico - geometric progression

Sol $\begin{vmatrix} a\alpha+b & a & b \\ b\alpha+c & b & c \\ 0 & a\alpha+b & b\alpha+c \end{vmatrix} = 0$

$R_3 \to R_3 - \alpha R_1 - R_2 \begin{vmatrix} a\alpha+b & a & b \\ b\alpha+c & b & c \\ -\left(a\alpha^2+2b\alpha+c\right) & 0 & 0 \end{vmatrix} = 0$

$\Rightarrow -\left(a\alpha^2+2b\alpha+c\right).\left(\alpha c-b^2\right)=0$

$\Rightarrow ac-b^2=0 \ \left(by \ \ a\alpha^2+2b\alpha+c \neq 0\right)$

56. If a, b, c, d $\in R$ are such that $a^2 + b^2 = 4$ and $c^2 + d^2 = 2$ and if $(a+ ib)^2 = (c + id)^2(x + iy)$ then $x^2 + y^2 =$

1) 4 2) 3 3) 2 4) 1

SOL: $|a+ib|^2 = |c+id|^2 |x+iy|$

$\left(\sqrt{a^2+b^2}\right)^2 = \left(\sqrt{c^2+d^2}\right)^2 \sqrt{x^2+y^2}$

$\left(a^2+b^2\right) = \left(c^2+d^2\right)\sqrt{x^2+y^2}$

$\left(a^2+b^2\right) = 2\sqrt{x^2+y^2}$

$\Rightarrow x^2+y^2=4$

57. If z is complex number such that $\left|z-\frac{4}{z}\right|=2$, then the greatest value of $|z|$ is

1) $1+\sqrt{2}$ 2) $\sqrt{2}$ 3) $\sqrt{3}+1$ 4) $1+\sqrt{5}$

SOL: $|z| = \left|z-\frac{4}{z}+\frac{4}{z}\right| \leq \left|z-\frac{4}{z}\right| + \left|\frac{4}{z}\right|$

$|z| \leq 2+\frac{4}{|z|}$ $\Rightarrow |z|^2 - 2|z| \leq 4 \Rightarrow \left(\left|z^2\right|-1\right)^2 \leq 5 \Rightarrow$ $|z| \leq \sqrt{5}-1$

58. If α is a non real root of the equation $x^6 - 1 = 0$ then $\frac{\alpha^2+\alpha^3+\alpha^4+\alpha^5}{\alpha+1}$

1) α 2) 1 3) 0 4) – 1

SOL: Since $\alpha^6 = 1$ w.t $1+\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5 = 0$

$(1+\alpha)+\alpha^2(1+\alpha)+\alpha^4(1+\alpha) = 0$

$(1+\alpha)(1+\alpha^2+\alpha^4) = 0$ G.P = $\frac{\alpha^2(1+\alpha)+\alpha^4(1+\alpha)}{1+\alpha} = \alpha^2+\alpha^4 = -1$ (key -4)

59. The minimum value of $27\tan^2\theta + 3\cot^2\theta$ is

1) 15 2) 18 3) 24 4) 30

SOL: Since A.M $\geq$ G.M

$$\frac{27\tan^2\theta+3\cot^2\theta}{2} \geq \sqrt{27\tan^2\theta.3\cot^2\theta}$$

= $27\tan^2\theta+3\cot^2\theta \geq 18$ (key - 2)

60. $\cos 36^0 - \cos 72^0$ =

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) $\frac{1}{8}$

SOL: $\text{Cos}36^0 - \text{Cos}72^0 = -2\sin 54^0.\sin(-18^0) = \frac{1}{2}$

61. $\tan x + \tan\left(x+\frac{\pi}{3}\right) + \tan\left(x+\frac{2\pi}{3}\right) = 3 \Rightarrow \tan 3x =$

1) 3 2) 2 3) 1 4) 0

SOL: $x = 15^0$ satisfy given equation

$\tan 3x = \tan 45^0 = 1$ (key = 3)

62. $3\sin x + 4\cos x = 5 \Rightarrow 6\tan\frac{x}{2} - 9\tan^2\frac{x}{2} =$

1) 0 2) 1 3) 3 4) 4

SOL: $3\left(\frac{2t}{1+t^2}\right)+4\left(\frac{1+t^2}{1+t^2}\right) = 5$ (where $t = \tan\frac{x}{2}$)

$6t - 9t^2 = 1$

$6\tan\frac{x}{2} - 9\tan^2\left(\frac{x}{2}\right) = 1$ (key - 2)

63. **If** $\frac{1}{2} \le x \le 1$ **then** $\cos^{-1}\left(\frac{x}{2}+\frac{1}{2}\sqrt{3-3x^2}\right) =$

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{3}$ 3) π 4) 0

SOL: $\mathrm{Cos}^{-1}x + \mathrm{Cos}^{-1}\left(\frac{x}{2}+\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right) =$

$\mathrm{Cos}^{-1}x + \mathrm{Cos}^{-1}\left(\frac{1}{2}\right) - \mathrm{Cos}^{-1}x = \frac{\pi}{3}$ (key - 2)

64. If a, b, c form a geometric progression with common ratio r, then the sum of the ordinates of the points of intersection of the line ax + by + c = 0 and the curve $x + 2y^2 = 0$ is

1) $-\frac{r^2}{2}$ 2) $-\frac{r}{2}$ 3) $\frac{r}{2}$ 4) r

SOL: a = a, b = ar, $c = ar^2$

line in $\cancel{a}x + \cancel{a}ry + \cancel{a}r^2 = 0$

$x + ry + r^2 = 0$

$x = -ry - r^2$ ------- (1)

sub in curve

$2y^2 - ry - r^2 = 0$

sum of ordinates = $\frac{r}{2}$

65. The point (3, 2) undergoes the following three transformations in the order given

i) Reflection about the line y = x

ii) Translation by the distance 1 unit in the positive direction of x – axis

iii) Rotation by an angle $\frac{\pi}{4}$ about the origin in the anticlockwise direction.

Then the final position of the point is

1) $\left(-\sqrt{18}, \sqrt{18}\right)$ 2) (–2, 3) 3) $\left(0, \sqrt{18}\right)$ 4) $(0, 3)$

SOL: Given point = (3, 2)

i) reflection about the line = (2, 3)

ii) translation through a distance = (3, 2)

iii) $X = -x\cos\theta + y\sin\theta = \left(\frac{-3}{\sqrt{2}}+\frac{3}{\sqrt{2}}\right) = 0$

$Y = +x\sin\theta + y\cos\theta = \left(\frac{3}{\sqrt{2}}+\frac{3}{\sqrt{2}}\right) = 3\sqrt{2}$

66. If X is a poission variate such that $\alpha = p(X=1) = P(X=2)$ then $P(X=4) =$

1) 2α 2) $\frac{\alpha}{3}$ 3) αe^{-2} 4) αe^{2}

SOL: P(X = 1) = P (X = 2)

$$\frac{\cancel{e^{-\lambda}}\,\cancel{\lambda}}{1!} = \frac{\cancel{e^{-\lambda}}\,\cancel{\lambda^2}}{2!} \qquad \lambda = 2$$

$$\alpha = P(X=1) = e^{-2}.2 = \frac{2}{e^2}$$

$$P(X=4) = \frac{e^{-2}.16}{24} = \frac{2}{3}e^{-2} = \frac{\alpha}{3}$$

67. Suppose X follows a binomial disttibution with parameters n and p, Where $0 < p < 1$. If $\frac{p(X=r)}{p(X=n-r)}$ is independent of n for every r, then p=

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{8}$

SOL: $\frac{P(X=r)}{P(X=n-r)} = \frac{\cancel{{}^nC_r}\, q^{n-r} p^r}{\cancel{{}^nC_{n-r}}\, q^r p^{n-r}} = \left(\frac{q}{p}\right)^{n-r}$ $\qquad \frac{q}{p} = 1 \;\; p = \frac{1}{2}$

68. In an entrance test there are multiple choice questions. There are four possible answers to each question, of which one is corret.The probability that a student know the answer to a question is 9/10.If he gets the correct answer to a question, then the probability that he was guessing is

1) $\frac{37}{40}$ 2) $\frac{1}{37}$ 3) $\frac{36}{37}$ 4) $\frac{1}{9}$

SOL: $\text{req. probability} = \frac{\frac{1}{10}.\frac{1}{4}}{\frac{9}{10}.1 + \frac{1}{10}.\frac{1}{4}} = \frac{1}{37}$

$$= \frac{9}{10} + \frac{1}{40} = \frac{37}{40} \Rightarrow \frac{\frac{1}{40}}{\frac{34}{40}} = \frac{1}{37}$$

69. **There are four machines and it is known that exactly two of them are faluty .They are tested one by one, in a random order till both the faulty machines are identifid. Then the probability that only two tests are need is**

1) $\frac{1}{3}$ 2) $\frac{1}{6}$ 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

SOL: $\text{Probability} = \frac{2}{4}.\frac{1}{3} + \frac{2}{4}.\frac{1}{3} = \frac{1}{3}$

70. **A fair coin is tossed 100 times. The probability of getting tails an odd number of times is**

1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{1}{8}$ 4) $\frac{3}{8}$

SOL: $\frac{2^{99}}{2^{100}} = \frac{1}{2}$

71. $\vec{a} = \vec{i} + \vec{j} - 2\vec{k} \Rightarrow \sum\{(\vec{a} \times \vec{i}) \times \vec{j}\}^2 =$

1) $\sqrt{6}$ 2) 6 3) 36 4) $6\sqrt{6}$

SOL: $\sum \left(\left(\bar{a} . j \right) i \right)^2 = \bar{a}^2 = 6$

72. **Let $\bar{a}, \bar{b}$ and $\bar{c}$ be three non-coplannar vectors and let $\bar{p}, \bar{q}$ and $\bar{r}$ be the vectors defined by**

$\bar{p} = \frac{\bar{b} \times \bar{c}}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}, \bar{q} = \frac{\bar{c} \times \bar{a}}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}, \bar{r} = \frac{\bar{a} \times \bar{b}}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}$ **. Then** $\left(\bar{a} + \bar{b}\right).\bar{p}\left(\bar{b} + \bar{c}\right).\bar{q}\left(\bar{c} + \bar{a}\right)\bar{r} =$

1) 0 2) 1 3) 2 3) 3

SOL: $\bar{a}.\bar{p} = \bar{b}.\bar{q} = \bar{c}.\bar{r} = 3$

73. **Let** $\bar{a} = \bar{i} + 2\bar{j} + \bar{k},\ \bar{b} = \bar{i} - \bar{j} + \bar{k},\ \bar{c} = \bar{i} + \bar{j} - \bar{k}$ **. A vector in the plane of $\bar{a}$ and $\bar{b}$ has projection $\frac{1}{\sqrt{3}}$ on $\bar{c}$. Then, one such vector is**

1) $4\bar{i} + \bar{j} - 4\bar{k}$ 2) $3\bar{i} + \bar{j} - 3\bar{k}$ 3) $4\bar{i} - \bar{j} + 4\bar{k}$ 4) $2\bar{i} + \bar{j} - 2\bar{k}$

SOL; By verification (3)

74. **The point if intersection of the lines**

$l_1 : \bar{r}\left(t\right) = \left(\bar{i} - 6\bar{j} + 2\bar{k}\right) + t\left(\bar{i} + 2\bar{j} + \bar{k}\right)$

$l_2 : \overline{R}\left(u\right) = \left(4\bar{j} + \bar{k}\right) + u\left(2\bar{i} + \bar{j} + 2\bar{k}\right)$ **is**

1) (4, 4, 5) 2) (6, 4, 7) 3) (8, 8, 9) 4) (10, 12, 11)

SOL; By verification (3)

75. **Thevectors** $\overline{AB} = \overline{3i} - \overline{2j} + \overline{2k}$ **and** $\overline{BC} = \bar{i} - \overline{2k}$ **are the adjacent sides of a parallelegram. The angle between its diagonals is**

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3} or \frac{2\pi}{3}$ 3) $\frac{3\pi}{4} or \frac{\pi}{4}$ 4) $\frac{5\pi}{6} or \frac{\pi}{6}$

SOL: $\overline{AC} = 2\bar{i} - 2\bar{j} + \bar{k}$

$\overline{BD} = -4i + 2j + 4\bar{k}$

Angle = $\cos^{-1}\left(\frac{\overline{AC}.\overline{BD}}{\left|\overline{AC}\right| \left|\overline{BD}\right|}\right) = \frac{\pi}{4} or \frac{3\pi}{4}$

76. If p^{th}, q^{th}, r^{th} terms of a geometric progression are the positive numbers a, b , c respectively, then the angle between the vectors $\left(\log a^2\right)\bar{i} + \left(\log b^2\right)\bar{j} + \left(\log c^2\right)\bar{k}$ and

$(q-r)\bar{i} + (r-p)\bar{j} + (p-q)\bar{k}$ is

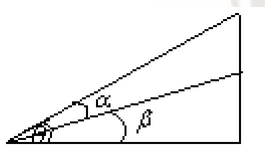
1) $\frac{\pi}{3}$ 2) $\frac{\pi}{2}$ 3) $\sin^{-1}\frac{1}{\sqrt{a^2+b^2+c^2}}$ 4) $\frac{\pi}{4}$

SOL: By verification, angle = $\frac{\pi}{2}$

77. A vertical pole subtends an angle $\tan^{-1}\left(\frac{1}{2}\right)$ at a point P on the ground. If the angles substended by the upper half and the lower half of the pole at P are respectively α and β, then $\left(\tan\alpha, \tan\beta\right) =$

1) $\left(\frac{1}{4}, \frac{1}{5}\right)$ 2) $\left(\frac{1}{5}, \frac{2}{9}\right)$ 3) $\left(\frac{2}{9}, \frac{1}{4}\right)$ 4) $\left(\frac{1}{4}, \frac{2}{9}\right)$

SOL;



$\tan\theta = \frac{1}{2}$

$\alpha + \beta = \theta$

$\tan\theta = \tan\left(\alpha+\beta\right) \Rightarrow \frac{1}{2} = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} \Rightarrow$ by verification $\tan\alpha = \frac{2}{9} \Rightarrow \tan\beta = \frac{1}{4}$

78. If α, β, γ are length of the altitudes of a traingle ABC with area Δ, then

$\frac{\Delta^2}{R^2}\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right) =$

1) $\sin^2 A + \sin^2 B + \sin^2 C$ 2) $\cos^2 A + \cos^2 B + \cos^2 C$

3) $\tan^2 A + \tan^2 B + \tan^2 C$ 4) $\cot^2 A + \cot^2 B + \cot^2 C$

SOL: $\Delta = \frac{1}{2}a\alpha = \frac{1}{2}b\beta = \frac{1}{2}c\gamma$

$= \alpha = \frac{2\Delta}{a}, \beta = \frac{2\Delta}{b}, \gamma = \frac{2\Delta}{c}$

G. P = $\frac{\Delta^2}{R^2}\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right) = \frac{\Delta^2}{R^2}\left(\frac{a^2+b^2+c^2}{4\Delta^2}\right)$

$= \frac{4R^2(\sin^2 A + \sin^2 B + \sin^2 C)}{4R^2}$

$= \sin^2 A + \sin^2 B + \sin^2 C$ (key - 1)

79. In an acute-angled traingle, $\cot B\cot C+\cot A\cot C+\cot A\cot B=$

1) -1 2) 0 3) 1 4) 2

SOL: Since A + B + C = π

A + B = $\pi - C$

Taking cot on b.s

= cot (A+B) = Cot (π -C)

$$\frac{\mathrm{Cot\,ACotB}-1}{\mathrm{CotB}+\mathrm{CotA}}=-\mathrm{CotC}$$

= Cot ACotB + CotB CotC + CotC CotA = 1 (key -3)

80. $x=\log\left(\frac{1}{y}+\sqrt{1+\frac{1}{y^2}}\right)\Rightarrow y=$

1) tan hx 2) cot hx 3) sechx 4) cosechx

SOL: $x=\log\left(\frac{1+\sqrt{1+y^2}}{y}\right)$

$x=\mathrm{cosech}^{-1}y$

$y=\mathrm{cosech}(x)$ (key - 4)

* * * * *

PHYSICS

81. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a rigid support. The time taken by the transverse wave formed in the rope to travel through the full length of the rope is (Assume g = $9.8\, m/s^2$)

1) 0.5 s 2) 1.6 s 3) 1.2 s 4) 1.0 s

Sol: [4]

$$t = 2\sqrt{\frac{l}{g}} = 2\sqrt{\frac{2.45}{9.8}}$$

= 1 sec

82. When a vibrating tuning fork is placed on a sound box of a sonometer, 8 beats per second are heard when the length of the sonometer wire is kept at 101 cm or 100 cm. Then the frequency of the tuning fork is (consider that the tension in the wire is kept constant)

1) 1616 Hz 2) 1608 Hz 3) 1632 Hz 4) 1600 Hz

Sol: [2]

$$n_1 l_1 = n_2 l_2$$

(n+8) 100 = (n-8) 101

$$\frac{n+8}{n-8} = \frac{101}{100}$$

$$\frac{n}{8} = \frac{201}{1}$$

n = 1608

83. The objective and eyepiece of an astronomical telescope are double convex lenses with refractive index 1.5. When the telescope is adjusted to infinity, the separation between the two lenses is 16 cm. If the space between the lenses is now filled with water and again telescope is adjusted for infinity, then the present separation between the lenses is

1) 8 cm 2) 16 cm 3) 24 cm 4) 32 cm

Sol: [4]

$$\frac{\mu_3}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} + \frac{\mu_3 - \mu_2}{R_2}$$

$$\frac{4}{3 f_0^1} - \frac{1}{\propto} = \frac{1.5 - 1}{R} + \frac{\frac{4}{3} - 1.5}{-R}$$

$$\Rightarrow f_0^1 = 2f$$

Now length = $f_0^1 + fe^1$

$= 2f_0 + 2fe$

= 2 (L)

= 2 (16)

= 32 cm

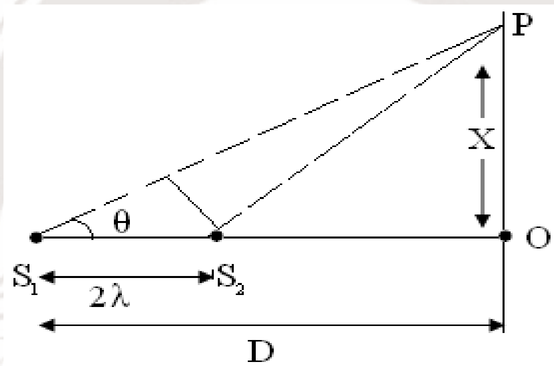
84. The dispersive powers of the materials of two lenses forming an achromatic combination are in the ratio of 4:3. Effective focal length of the two lenses is +60 cm then the focal lenghts of the lenses should be

1) - 20 cm, 25 cm 2) 20 cm, - 25 cm 3) - 15 cm, 20 cm 4) 15 cm, - 20 cm

Sol: [4]

ratio is 3:4 and convex lens has small focal length

85. Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between them is 2 λ as shown in figure. The first bright fringe is formed at 'P' due to interference on a screen placed at a distance 'D' from S_1 (D >> λ), then OP is



1) $\sqrt{2}$D 2) 1.5 D 3) $\sqrt{3}$D 4) 2 D

Sol: $(S_1P)^2 = D^2 + x^2$

$(S_2P)^2 = (D-2\lambda)^2 + x^2$

$(S_1P)^2 - (S_2P)^2 = D^2 + x^2 - [D^2 + 4\lambda^2 - 4D\lambda + x^2]$

$= 4D\lambda - 4\lambda^2$

$2S_1P(S_1P - S_2P) = 4D\lambda$

$2(S_1P)(\lambda) = 4D\lambda$

$S_1P = 2D$

$\sqrt{D^2 + x^2} = 2D$

$D^2 + x^2 = 4D^2$

$x^2 = 3D^2$

$x = \sqrt{3}D$

Method-2

$2\lambda\cos\theta = \lambda$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^0 \quad \tan\theta = \frac{x}{D}$

$\sqrt{3} = \frac{x}{D} \Rightarrow x = \sqrt{3}D$

86. A short bar magnet in a vibrating mangnetometer makes 16 oscillations in 4 seconds. Another short magnet with same length and width having moment of inertia 1.5 times the first one is placed over the first magnet and oscillated. Neglecting the induced magnetization, the time period of the combination is

1) $2\sqrt{10}$ s 2) $20\sqrt{10}$ s 3) $\frac{2}{\sqrt{10}}$ s 4) $\frac{2.5}{\sqrt{10}}$ s

Sol: Bonus

87. A magnetic needle lying parallel to a magnetic field is turned through 60^0. The work doen on it is w. The torque required to maintain the magnetic needle in the position mentioned above is

1) $\sqrt{3}$ w 2) $\frac{\sqrt{3}}{2}$ w 3) w/2 4) 2 w

Sol: [1]

$$w = MB(1-\cos 60)$$

$$= \frac{MB}{2}$$

$$\tau = MB\sin\theta$$

$$= MB\frac{\sqrt{3}}{2}$$

$$= \sqrt{3}\, w$$

88. A parallel plate capacitor has a capacity 80×10^{-6} F when air is present between the plates. The volume between the plates is then completely filled with a dielectric slab of dielectric constant 20. The capacitor is now connected to a battery of 30 V by wires. The dielelctric slab is then removed. Then, the charge that passes now through the wire is

1) $45.6\times10^{-3}C$ 2) $25.3\times10^{-3}C$ 3) $120\times10^{-3}C$ 4) $120\times10^{-3}C$

Sol: [1]

$$\Delta q = \Delta c V$$

$$= \left(c^1 - c\right)V$$

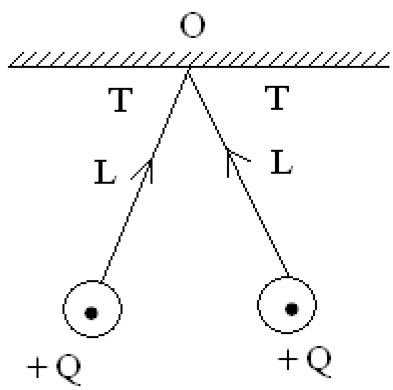
$$= (k-1)\sigma V$$

$$= (20-1)\left(80\times10^{-6}\right)(30)$$

$$= 4.56\times10^{-2}$$

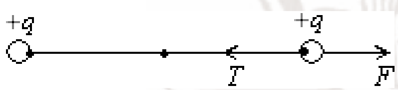
$$= 45.6\times10^{-3}C$$

89. Two small spheres each having equal positive charge Q (Couloumb) on each are suspended by two insulating strings of equal length L (meter) from a rigid hook (shown in Fig.). The whole set up is taken into satellite where there is no gravity. The two balls are now held by elelctrostatic forces in horizontal position, the tension in each string is then



1) $\frac{Q^2}{16\pi\varepsilon_0 L^2}$ 2) $\frac{Q^2}{8\pi\varepsilon_0 L^2}$ 3) $\frac{Q^2}{4\pi\varepsilon_0 L^2}$ 4) $\frac{Q^2}{2\pi\varepsilon_0 L^2}$

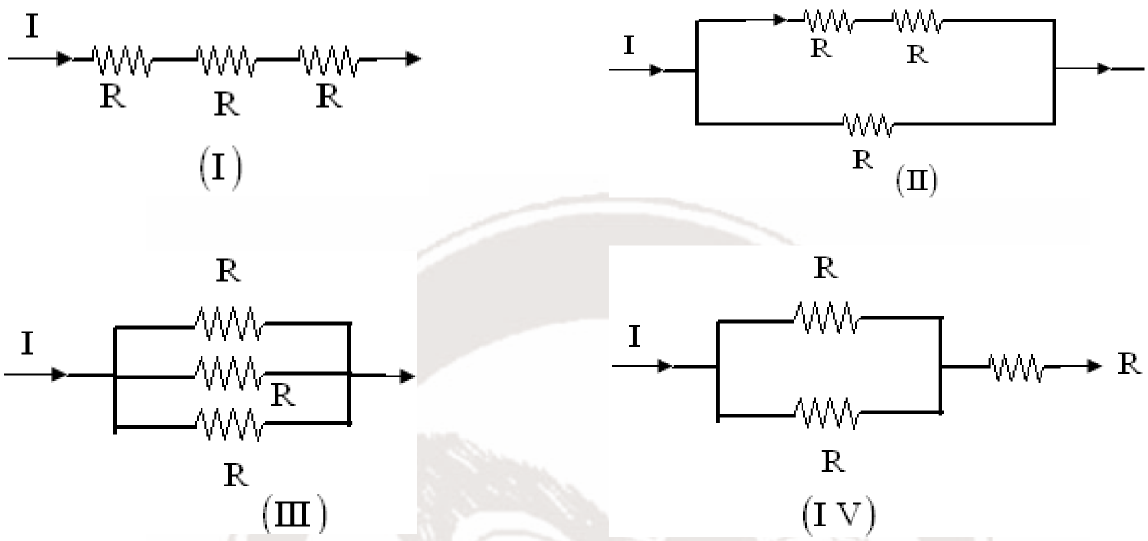
Sol: [1]



$T = F$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q^2}{(2L)^2} = \frac{Q^2}{16\pi\varepsilon_0 L^2}$$

90. Three resistances of equal values are arranged in four different configurations as shown below. Power dissipation in the increasing order is



1) (III) < (II) < (IV) < (I) 2) (II) < (III) < (IV) < (I)

3) (I) < (IV) < (III) < (II) 4) (I) < (III) < (II) < (IV)

Sol: [1]

I - $I^2(3R)$

II - $I^2\left(\frac{2R}{3}\right)$

III - $I^2\left(\frac{R}{3}\right)$

IV - $I^2\left(\frac{3}{2}R\right)$

Hence, option

91. Four resistors A,B,C and D form a Wheatstones bridge. The bridge is balanced when C = 100 Ω. If A and B are interchanged, the bridge balances for C= 121 Ω. The value of D is

1) 10 Ω 2) 100 Ω 3) 110 Ω 4) 120 Ω

Sol: [3]

$$\frac{A}{B} = \frac{100}{x}$$

$$\frac{B}{A} = \frac{|2|}{x}$$

$$\therefore \frac{100}{x} = \frac{x}{|2|}$$

$$x^2 = 12100$$

$$x = 110\ \Omega$$

92. Total emf produced in a thermocouple does not depend on

1) the metals in the thermocouple

2) thomson coefficients of the metals in the thermocouple

3) temperature of the junctions

4) the duration of time for which the current is passed through thermocouple

Sol: [4]

93. A long curved conductor carries a current $\overline{\mathrm{I}}$ $\left(\overline{\mathrm{I}}\text{ is a vector}\right)$. A small current element of length $\overline{\mathrm{d}l}$, on the wire induces a magnetic field at a point, away from the current element. If the position vector between the current element and the point is $\overline{\mathrm{r}}$, making an angle with current element then, the induced magnetic field density; $\overline{\mathrm{dB}}$ (vector) at the point is (μ_0 = permeability of free space)

1) $\dfrac{\mu_0 \mathrm{I}\overline{\mathrm{d}l}\times\overline{\mathrm{r}}}{4\pi r}$ perpendicular to the current element $\overline{\mathrm{d}l}$

2) $\dfrac{\mu_0 \overline{\mathrm{I}}\times\overline{\mathrm{r}}\times\overline{\mathrm{d}l}}{4\pi r^2}$ perpendicular to the current element $\overline{\mathrm{d}l}$

3) $\dfrac{\mu_0 \overline{\mathrm{I}}\times\overline{\mathrm{d}l}}{r}$ perpendicular to the plane containing the current element and position vector $\overline{\mathrm{r}}$

4) $\dfrac{\mu_0 \overline{\mathrm{I}}\times\overline{\mathrm{d}l}}{4\pi r^2}$ perpendicular to the plane containing current element and position vector $\overline{\mathrm{r}}$

Sol: Bonus

94. A primary coil and secondary coil are placed close to each other. A current, which changes at the rate of 25 amp in a millisecond, is present in the primary coil. If the mutual inductance is 92×10^{-6} Henries, then the value of induced emf in the secondary coil is

1) 4.6 V   2) 2.3 V   3) 0.368 mV   4) 0.23 mV

Sol: [2]

$$e = M\frac{di}{dt} \quad = 92\times10^{-6}\times\frac{25}{1\times10^{-3}} = 2.3V$$

95. The de Broglie wavelength of an electron moving with a velocity of 1.5×10^{8} m/s is equal to that of a photon. The ratio of kinetic energy of the electron to that of the photon $\left(C = 3\times10^{8}\,m/s\right)$

1) 2   2) 4   3) $\frac{1}{2}$ 4) $\frac{1}{4}$

Sol: [4]

$$Ratio = \frac{v}{2C}$$

$$= \frac{1.5\times10^{8}}{2\times3\times10^{8}} = \frac{1}{4}$$

96. A proton when accelerated thrugh a potential differnce of V, has a de Broglie wavelength λ associated with it. If an alpha particle is to have the same de Broglie wavelength λ, it must be accelerated through a potential difference of

1) $\frac{V}{8}$ 2) $\frac{V}{4}$ 3) 4 V 4) 8 V

Sol: [1]

$\lambda_p = \lambda_\alpha$

$(mqv)_p = (mqv)_\alpha$

$v_\alpha = \frac{vp}{8}$

97. The half life of Ra^{226} is 1620 years. Then the number of atoms decay in one second in 1 gm of radium (Avogadro number = 6.023 $\times$ 10^{23})

1) 4.23×10^{9} 2) 3.16×10^{10} 3) 3.61×10^{10} 4) 2.16×10^{10}

Sol: [3]

$\frac{dN}{dt} = \lambda N$

$= \frac{0.693}{1620\times365\times86\times400}\times\frac{6.023\times10^{23}}{226}$

$= 3.61\times10^{10}$

98. The half life of a radioactive element is 10 hours. The fraction of initial radioactivity of the element that will remain after 40 hours is

1) $\frac{1}{2}$ 2) $\frac{1}{16}$ 3) $\frac{1}{8}$ 4) $\frac{1}{4}$

Sol: [2]

After 4 half lifes

$\frac{N_0}{2^4} = \frac{N_0}{16}$

99. In a transistor if $\frac{I_C}{I_E} = \alpha$ and $\frac{I_C}{I_B} = \beta$. If α varies between $\frac{20}{21}$ and $\frac{100}{101}$, then the value of β lies between

1) 1-10 2) 0.95-0.99 3) 20-100 4) 200-300

Sol: [3]

$\beta = \frac{\alpha}{1-\alpha}$

$\beta_1 = \frac{20/21}{1-\frac{20}{21}} = 20$ $\beta_2 = \frac{100/101}{1-\frac{100}{101}} = 100$

100. Match column A (layers in the ionosphere for skywave propagation) with column B (their height range) :

Column A	Column B
I) D-layer	a) 250-400 km
II) E-layer	b) 170-190 km
III) F_1 -layer	c) 95-120 km
IV) F_2 -layer	d) 65-75 km

The corrent answer is

	I	II	III	IV		I	II	III	IV
1)	a	b	c	d	2)	d	c	a	b
3)	d	c	b	a	4)	c	d	c	b

Sol: [3]

101. The gravitational field in a region is given by equation $\vec{E}=(5\hat{i}+12\hat{j})N/kg$. If a particle of mass 2 kg is moved from the origin to the point (12m,5m) in this region, the change in gravitational potential energy is

1) -225 J 2) -240 J 3) -245 J 4) -250 J

Sol: [2]

$$dv=-E.dr$$

$$=-(5\hat{i}+12\hat{j}).(12\hat{i}+15\hat{j}) \quad =(60+60)=-120$$

$$U=mdv=2\times(-120)=-240J$$

102. The time period of a particle in simple harmonic motion is 8s. At t=o, it is at the mean position. The ratio of the distances travelled by it in the first and second seconds is

1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{1}{\sqrt{2}-1}$ 4) $\frac{1}{\sqrt{3}}$

Sol: [3]

$$y_1=A\sin\frac{2\pi}{8}\times 1=\frac{A}{\sqrt{2}} \qquad y_2=A-\frac{A}{\sqrt{2}} \qquad \frac{y_1}{y_2}=\frac{1}{\sqrt{2}-1}$$

103. A tension of 22 N is applied to a coppper wire of cross-sectional area 0.02 cm^2 Young's modulus of copper is $1.1\text{x}10^{11}$ N/m^2 and poisson's ratio 0.32. The decrease in cross sectional area will be

1) $1.28\times10^{-6}cm^2$ 2) $1.6\times10^{-6}cm^2$ 3) $2.56\times10^{-6}cm^2$ 4) $0.64\times10^{-6}cm^2$

Sol: [4]

$$y=\frac{F\times l}{Ae}; \quad e/l=\frac{F}{Ay}=\frac{22}{0.02\times10^{-4}\times1.1\times10^{11}}=10^{-4}$$

$$\sigma=\frac{-\frac{\Delta r}{r}}{\frac{\Delta r}{r}}; \quad \frac{\Delta r}{r}=\sigma\frac{\Delta l}{l}=0.32\times\frac{\Delta l}{l}=0.32\times10^{-4}=32\times10^{-6}$$

$$\frac{1}{2}\frac{\Delta A}{A}=0.32\times e/l \qquad \Delta A=0.64\times10^{-6}cm^2$$

104. Drops of liquid of density ‘d’ are floating half immersed in a liquid of density ρ. if the surface tension of the liquid is T, then the radius of the drop is

1) $\sqrt{\dfrac{3T}{g(3d-\rho)}}$ 2) $\sqrt{\dfrac{6T}{g(2d-\rho)}}$ 3) $\sqrt{\dfrac{3T}{g(2d-\rho)}}$ 4) $\sqrt{\dfrac{3T}{g(4d-3\rho)}}$

Sol: [3]

$$\frac{4}{3}\pi r^3 dg = \frac{2}{3}\pi r^3 \rho g + T \times 2\pi r$$

$$r = \sqrt{\frac{3T}{g\left(2d-\rho\right)}}$$

105. A pipe having an internal diameter ‘D’ is connected to anther pipe of same size. water flows into the second pope through ‘n’ holes, each of diameter ‘d’. If the water in the first pipe has speed ‘V’, the speed of water leaving the second pipe is

1) $\dfrac{D^2 v}{nd^2}$ 2) $\dfrac{nD^2 v}{d^2}$ 3) $\dfrac{nd^2 v}{D^2}$ 4) $\dfrac{d^2 v}{nd^2}$

Sol: [1]

$$\pi\left(\frac{D}{2}\right)^2 X\vartheta = \left(\frac{d}{2}\right)^2 X\vartheta^1 \qquad v^1 = \frac{D^2 v}{nd^2}$$

106. When a liquid is heated in copper vessel its Coefficient of apparent expansion is $6x10^{-6}/^0C$. When the same liquid is heated in a steel vesel its coefficient of apparent expansion is $24x10^{-6}/^0C$.. If coefficient of linear expansion for copper is $18x10^{-6}/^0C$. the coefficient of linear expansion for steel is

1) $20x10^{-6}/^0C$ 2) $24x10^{-6}/^0C$ 3) $36x10^{-6}/^0C$ 4) $12x10^{-6}/^0C$

Sol: [4]

$$\gamma_{a_1} + 3\alpha_1 = \gamma_{a_2} + 3\alpha_2$$

$$6\times10^{-6} + 3(18\times10^{-6}) = 24\times10^{-6} + 3\alpha_2$$

$$\alpha_2 = 12\times10^{-6}/^oC$$

107. When the temperature of a body increases from T to T + $\triangle$ T, its moment of inertia increases from I to I + $\triangle$ I. If α is the coefficient of linear expansion of the material of the body, then

$\dfrac{\triangle I}{I}$ is (neglect higher orders of α

1) $\alpha\triangle T$ **2)** $2\alpha\triangle T$ **3)** $\dfrac{\triangle T}{\alpha}$ **4)** $\dfrac{2\alpha}{\triangle T}$

Sol: [2]

$$\frac{\triangle I}{I} = 2\frac{\triangle k}{k} = 2\alpha\triangle T \qquad \frac{\triangle I}{I} = 2\alpha\triangle T$$

108. A sound wave passing through an ideal gas at NTP produces a pressure change of 0.001

dyne/cm2 during adiabatic compression. The corresponding change in temperature (γ =1.5 for the gas and atmospheric pressure is 1.013 x10^6 dynes/cm^2) is

1) $8.97X10^{-4}K$ 2) $8.97X10^{-6}K$ 3) $8.97X10^{-8}K$ 4) $8.97X10^{-9}K$

Sol: [3]

$T^{\gamma}p^{1-\gamma} = Const$

$T^{\gamma=}P^{\gamma-1}$

$T = (p)\frac{\gamma-1}{\gamma}$ $\frac{\triangle T}{T} = \frac{\gamma-1}{\gamma}\times\frac{\triangle p}{p}$

109. Work done to increase the temperature of one mole of an ideal gas by 30^0C, if it is expanding under the condition $V \propto T^{2/3}$ is, (R =8.314 J/mole/^{0} K)

1) 116.2J 2) 136.2J 3) 166.2J 4) 186.2J

Sol: [3]

$v\alpha T^{2/3}$

$P\alpha\sqrt{v}$

$P = k\sqrt{v}$

$w = \int p\delta v$

$w = \int kv^{3/2}$

$\frac{= kv^{3/2}}{3/2}$

$2/3\frac{P}{\sqrt{v}}v^{3/2} = 2/3pv$

= $\left[2/3NRT\right]_{T_1}^{T^2}$

= $2/3X11X8.3X30$

=166.2

110. Power radiated by a black body at temperature T_1 is P and it radiates maximum energy at a wavelength λ_1 .If the temperature of the black body is changed from T_1 to T_2 , it radiates maximum energy at a wavelength $\frac{\lambda_1}{2}$ The power radiated at T_2 is

1) 2 P 2) 4 P 3) 8 P 4) 16 P

Sol: [4]

$P\alpha T^4\alpha 1/\lambda^4$ $\frac{p_1}{p_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4$ $\frac{P_1}{P_2} = \frac{1}{16}$ $P_2 = 16P_1 = 16P$

111. Two solid spheres A and B each of radius 'R' are made of materials of densities ρ_A and ρ_B respectively. their moments of inertia about a diameter are I_A and I_B respectively. The value of $\frac{I_A}{I_B}$ is

1) $\sqrt{\frac{\rho_A}{\rho_B}}$ 2) $\sqrt{\frac{\rho_B}{\rho_A}}$ 3) $\frac{\rho_A}{\rho_B}$ 4) $\frac{\rho_B}{\rho_A}$

Sol: [3]

$$\frac{I_A}{I_B} = \frac{2/5 M_A R_H{}^2}{2/5 M_B R^2}$$

$$\frac{4/3\pi R^3 P_A}{4/3\pi R^3 p_{\mathbf{B}}}$$

$$= \frac{\rho_A}{\rho_B}$$

112. Assertion (A) : The moment of inertia of a steel sphere is larger than the moment of inertia of a wooden sphere of same radius.

Reason (R) : Moment of inertia is independent of mass of the body

The correct one is

1) Both (A) and (R) are true, and (R) is the correct explanation of (A)

2) Both (A) and (R) are true, and (R) is not the correct explanation of (A)

3) (A) is true but (R) is wrong

4) is wrong but (R) is true

113. When the engine is switched off a vehicle of mass 'M' is moving on a rough horizontal road with momentum P. If the coefficient of friction between the road and tyres of the vehicle is $\mu_{k,}$ the distance travelled by the vehicle before it comes to rest is

1) $\frac{p^2}{2\mu_k M^2 g}$ 2) $\frac{2\mu_k M^2 g}{P^2}$ 3) $\frac{P^2}{2\mu_k g}$ 4) $\frac{P^2 M^2}{2\mu_k g}$

Sol: [1]

$$\mu_k mgs = \frac{p^2}{2M}$$

$$S = \frac{p^2}{2M^2 \mu_k g}$$

114. Choose correct statement

(A) The position of centre of mass of a system is dependent on the choice of co-ordinate system.

(B) Newton's second law of motion is applicable to the centre of mass of the system

(C) Internal forces can change the state of centre of mass.

(D) Internal forces can change the state of centre of mass.

1) Both (A) and (B) are correct
2)Both (B) and (C) are wrong
3)Both (A) and (C) are wro ng
4)Both (A) and (D) are wrong

Sol: [4]

115. A ball 'A'of mass 'm' moving along positive x-direction with kinetic energy 'K' and momentum P undergoeselastic head on collision with a stationary ball B of mass 'M' After collistion the ball A moves along negative X-direction with kinetic energy $\frac{K}{9}$,Final momentum of B is

1) P
2) $\frac{P}{3}$
3) $\frac{4P}{3}$
4) 4P

Sol: [3]

$$K = \frac{1}{2}mu_1^2$$

$$u_1 = \sqrt{\frac{2k}{m}}$$

$$\frac{1}{2}mv_1^2 = \frac{k}{9}$$

$$v_1^2 = \frac{2k}{9m}$$

$$v_1 = \sqrt{\frac{2k}{9m}}$$

$$k = \frac{p^2}{2m}$$

$$mu_1 = -mv_1 + P_B$$

$$m\left(u_1 + v_1\right) = P_B$$

$$P_B = m\left[\sqrt{\frac{2k}{m}} + \sqrt{\frac{2k}{9m}}\right]$$

$$= \sqrt{2mk} + \sqrt{\frac{2mk}{3}}$$

$$= P + \frac{P}{3}$$

$$= \frac{4p}{3}$$

116. In Atwood's machine, two masses 3 kg and 5 kg are connected by a light string which passes over a frictionless pulley. The support of the pulley is attached to the ceiling of a compartment of a train. If the train moves in a horizontal direction with a constant acceleration 8 ms^{-2} the tension in the string in Newtons is $(g = 10ms^{-2})$

1) 3.75 2) 7.5 3) 15 4) 20

Sol: Bonus

$$a = \frac{20}{8} = \frac{5}{2}$$
$$= 2\sqrt{my^{-2}}$$
\
$$= Tan\theta = \frac{24}{2.5}$$

$$a = \sqrt{64 + 2.5}$$
$$= \sqrt{64 + 6.25}$$
$$= \sqrt{70.25}$$

117. The velocity 'v' reached by acar of mass 'm' at certain distance from the starting poing driven with constant power 'P' is such that

1) $v\alpha \frac{3P}{m}$ 2) $v^2\alpha \frac{3P}{m}$ 3) $v^3\alpha \frac{3P}{m}$ 4) $v\ \alpha\left(\frac{3P}{m}\right)^2$

Sol: [3]

$$p = \frac{1/2mv^2}{t}$$

118. It is possibleto project a particle with a given velocity in tow possible ways so as to make them pass through a poing p at a horizontal distance r from the point of projection. if t_1 and t_2 are times taken to rach this poing in two possible ways, then the product t_1t_2 is proportional to

1) $\frac{1}{r}$ 2) r 3) r^2 4) $\frac{1}{r^2}$

Sol: [2]

$$T_1 = \frac{2u\sin\theta}{g}$$

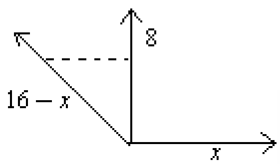
$$T_2 = \frac{2u\cos\theta}{g}$$

$$T_1T_2 = \frac{2u^2\sin\theta\cos\theta}{g}, = \frac{2r}{g}$$

119. Sum of magnitudes of two forces acting at a point is 16N. if their resultant is normal to smaller force, and has a magnitude 8 N, then forces are

1) 6N,10N 2) 8N,8N 3) 4N,12N 4) 2N,14N

Sol: [1]



$$(16-x)^2 = 8^2 + x^2$$

$$256 + x^2 - 32x = 64 + x^2$$

$$32x = 192$$

$$x = \frac{192}{32} = 6$$

$\therefore$ forces 6N, 10N

120. The length of a pendulum is measured as 1.01 m and time for 30 oscillations is measured as one minute 3 seconds. Error iength is 0.01 m and error in time is 3 secs. The percentage error in the measurement of acceleration due to gravity is

1) 1 2) 5 3)10 4)15

Sol: [3] $\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$

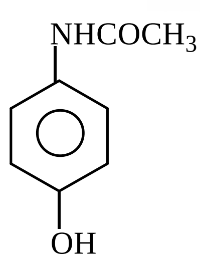
CHEMISTRY

121. 4-Hydroxy acetanilide belongs to which of the following ?

1) Antipyretic 2) Antacid 3) Antiseptic 4) Antihistamine

Ans : [1]

Sol : Benzene - 4-hydroxyacetanilide is an Antipyretic (theory)



122. The site of action of insulin is

1) Mitochondria 2) Nucleus 3) Plasma membrane 4) DNA

Ans : [3]

Sol : The site of action of Insulin is Plasma membrane (Theory)

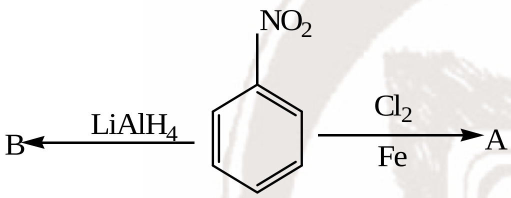
123. The monomer of neoprene is

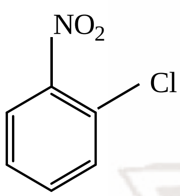
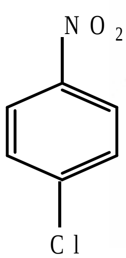
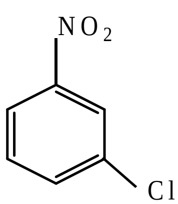
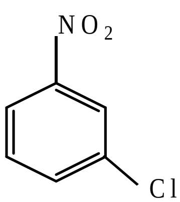
1) 1,3-Butadiene
2) 2-Chloro-1,3-butadiene
3) 2-Methyl-1,3-butadiene
4) Vinyl chloride

Ans : [2]

Sol :(Theory) $\overset{1}{CH_2} = \underset{\substack{|\\ Cl}}{\overset{2}{C}} - \overset{3}{CH} = \overset{4}{CH_2}$ 2-chloro-1,3-butadiene

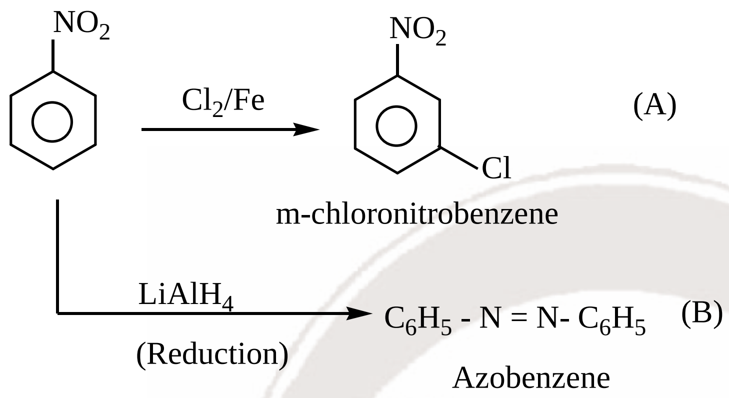
124. Identify A and B in the following reactions



A	**B**
1) 	$C_6H_5 - \underset{\substack{|\\ H}}{N} - \underset{\substack{|\\ H}}{N} - C_6H_5$
2) 	$C_6H_5 - N = N - C_6H_5$
3) 	$C_6H_5 - N = N - C_6H_5$
4) 	$C_6H_5 - NH_2$

Ans : [3]

Sol : Reactions of Nitrobenzene .(Theory)



125. What is the product obtained in the reaction of Acetaldehyde with semicarbazide ?

1) $H_3C-CH=N-NH-\overset{\overset{O}{\|}}{C}-NH_2$

2) $H_3C-CH=N-NH_2$

3) $H_3C-CH=N-OH$

4) $H_3C-\underset{\underset{CH_3}{|}}{C}=N-NH-\overset{\overset{O}{\|}}{C}-NH_2$

Ans : [1]

Sol : (Theory) Reaction of Acetaldehyde

$$CH_3-\underset{\underset{O}{\|}}{C}-H + NH_2-NH-CONH_2 \longrightarrow$$

$$CH_3-CH=N-NH-CONH_2$$

Acetaldehyde semicarbazone + H_2O

126. Compound -A (C_3H_6O) undergoes following reactions to form B and C . Identify A, B and C

$$C \xleftarrow{\text{Zn- Hg/HCl}} \underset{A}{C_3H_6O} \xrightarrow{I_2/NaOH} B$$

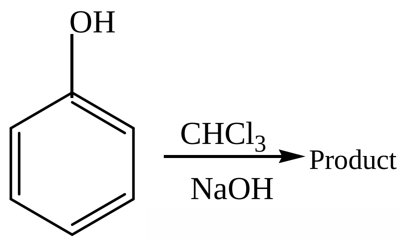
	A	B	C
1)	$H_3C-C(=O)-CH_3$	CHI_3	$H_3C-CH_2-CH_3$
2)	$H_3C=C(H)-CH_2OH$	CH_3I	$H_3C-CH_2-CH_2-OH$
3)	H_3C-CH_2-CHO	CH_3I	$H_3C-CH(OH)-CH_3$
4)	$H_3C-C(=O)-CH_3$	CHI_3	$H_3C-CH(OH)-CH_3$

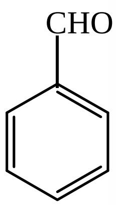
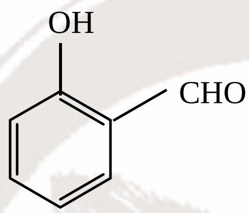
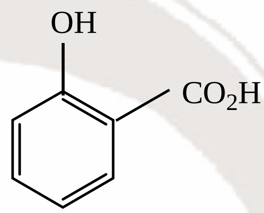
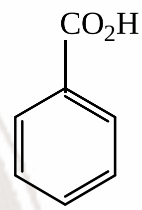
Ans : [1]

Sol : Theory (Reactions of Acetone)

$$\underset{\substack{\text{Propane}\\(C)}}{CH_3-CH_2-CH_3} \xleftarrow[\text{Clammensen's reduction}]{\text{Zn-Hg/conHcl}} \underset{\substack{\text{Acetone}\\(A)}}{CH_3COCH_3} \xrightarrow[\text{Halofrm Reaction}]{I_2/NaOH} \underset{\substack{(B)\\\text{Iodo form}}}{CHI_3}$$

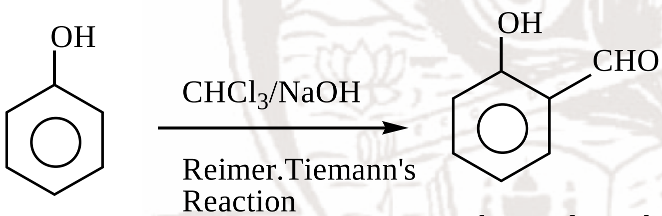
127. Identify the product in the following reaction



1)  2)  3)  4) 

Ans : [2]

Sol : Theory (Reaction of Phenol)



Ortho-Hydroxy benzaldehyde
(Salicylaldehyde)

128. With respect to chlorobenzene , which of the following statements is NOT correct ?

1) Cl is ortho/para directing 2) Cl exhibits +M effect

3) Cl is ring deactivating 4) Cl is meta directing

Ans : [4]

Sol : Theroy (Directional Nature of chlorine)

Cl- is not a metadirecting group

Cl-is an ortho, para directing group

Cl- is an electron with drawing group (-I group)

Cl- is a +M group (due to one pairs of electron of chlorine)

129. Match the following

List -I	List -II
A) Acetaldehyde, Vinylalcohol	**I) Enantiomers**
B) Eclipsed and staggered ethane	**II) Tautomers**
C) (+)2-Butanol, (-) 2- Butanol	**III) Chain isomers**
D) Methyl -n-propylamine and Diethylamine	**IV) Conformational isomers**
	V) Metamers

	(A)	**(B)**	**(C)**	**(D)**
1)	(II)	(IV)	(III)	(V)
2)	(II)	(IV)	(I)	(V)
3)	(V)	(I)	(IV)	(II)
4)	(V)	(I)	(III)	(II)

Ans : [2]

Sol : Theory

A) Acetaldehyde, Vinylalcohol - Tautomers(II)

B) Eclipsed and staggered ethane - Conformational isomers

C) (+)2-Butanol, (-) 2- Butanol - Enantiomers(I)

D) Methyl -n-propylamine and Diethylamine - Metamers

130. Which of the following statements is NOT correct ?

1) The six carbons in benzene are sp^2 hybridised

2) Benzene has $(4n+2)\pi$ electrons

3) Benzene undergoes substitution reactions

4) Benzene has two carbon-carbon bond lengths, $1.54\overset{0}{A}$ *and* $1.34\overset{0}{A}$

Ans : [4]

Sol :Theory

In Benzene C-C bond length is 1.39 A^0 due to resonance

131. Different conformations of the same molecule are called

1) Isomers 2) Epimers 3) Enantiomers 4) Rotamers

Ans : [4]

Sol : Theory . The other name of confomratoinal isomerism is Rotamerism and the different comfomations are called Rotamers

132. The chlorination of ethane is an example for which type of the following reactions ?

1) Nucleophilic substitution 2) Electrophilic substitution

3) Free radical substitution 4) Rearrangement

Ans : [3]

Sol : Theory . Chlorination of ethane in presence of sunlight is an example for free radical substitution reaction.

133. The pair of gases responsible for acid rain are

1) H_2, O_3 2) H_4C, O_3 3) NO_2,SO_2 4) CO, CH_4

Ans : [3]

Sol : Theory . The cause of Acid rain in due to oxides of Nitrogen and sulphur (NO_2 and SO_2)

134. In photoelectric effect, if the energy required to overcome the attractive forces on the electron, (work functions) of Li, Na and Rb are 2.41eV, 2.30eV and 2.09eV respectively, the work function of 'K' could approximately be in eV

1) 2.52 2) 2.20 3) 2.35 4) 2.01

Ans : [2]

Sol : (Theory)In IA group from Li to Cs (Top to bottom) IP value decreases. Therefore the energy required to overcome the attractive forces on the electron in potassium should be less than sodium and more than Rb

Li = 2.41ev Na = 2.30eV K= 2.20eV Rb=2.09eV

135. The quantum number which explains the line spectra observed as doublets in case of hydrogen and alkali metals and doublets and triplets in case of alkaline earth metals is

1) Spin 2) Azimuthal 3) Magnetic 4) Principal

Ans : [1]

Sol : Theory Principle , Azimuthal and magnetic quantum number are not enough to explain the line spectra observed as doublets in case of hydrogen and akali metals and doublets and triplets in the case of alkaline earth metals. This suggests the presence of fourth quantum number spin quantum number

136. Which one of the following cannot form an amphoteric oxide ?

1) Al 2) Sn 3) Sb 4) P

Ans : [4]

Sol : Theory .Phosphorus is a non.metal .Therefore it forms only Acidic oxide

137. The formal charges of C and O atoms in CO_2 ($:\ddot{O}=C=\ddot{O}:$) are, respectively

1) 1,-1 2) -1,1 3) 2,-2 4) 0,0

Ans : [4]

Sol : Formal charge = $N_A - N_{LP} - \frac{1}{2}N_{BP}]$

$_{A}$ = No.of electrons in the valency shall in the free atom

N_{Lp} = No.of electrons in line pairs

N_{BP} = No.of electrons in Bond pairs

$$:\ddot{O}=C=\ddot{O}:$$

$$Qf = 6-4-\frac{1}{2}(4)=0$$

$$Qf = 6-4-\frac{1}{2}(4)=0$$

138. According to molecular orbital theory, the total number of bonding electron pairs in O_2 is

1) 2 2) 3 3) 5 4) 4

Ans : [3]

Sol : $\boxed{\sigma 1s^2}\sigma^* 1s^2 \boxed{\sigma 2s^2}\ \sigma^* 2s^2\ \boxed{\sigma 2p_z^2}$

$\boxed{\pi 2P_4^2} = \boxed{\pi 2Px^2}\ \pi^* 2p_x^1 = \pi * 2p_y^1$

139. One mole of N_2H_4 loses 10 moles of electrons to form a new compound Z. Assuming that all the nitrogens appear in the new compound, what is the oxidation state of nitrogen in Z? (There is no change in the oxidation state of hydrogen)

1) -1 2) -3 3) +3 4) +5

Ans : [3]

Sol : $\overset{-2}{N_2}\ \overset{+1}{H_4}$

$$\overset{-2}{N_2}\ \overset{+1}{H_4} \rightarrow \begin{cases} N^{+3} + 5e^- \\ N^{+3} + 5e^- \\ (Z) \end{cases}$$

140. Which one of the following equations represents the variation of viscosity coefficient (η) with temperature (T) ?

1) $\eta = Ae^{-E/RT}$ **2)** $\eta = Ae^{E/RT}$ **3)** $\eta = Ae^{-E/kT}$ **4)** $\eta = Ae^{-E/T}$

Ans : [2]

Sol : Theory

$\eta = A.e^{E/RT}$

141. The weight in grams of a non-volatile solute (M.wt:60) to be dissolved in 90 g of water to produce a relative lowering of vapour pressure of 0.02 is

1) 4 2) 8 3) 6 4) 10

Ans : [3]

Sol : Numerical

$$\frac{P^0 - P}{P^0} = x_2 = \frac{w}{m} \times \frac{M}{W}$$

w=wt of solute

m= mol wt of solute

W = wt of solvent

M= molut solvent

x_2= Mole fraction of solute

$$0.02 = \frac{w}{60} \times \frac{18}{90}$$

$$w = 6gm$$

142. The experimentally determined molar mass of a non-volatile solute, $BaCl_2$ in water by Cottrell's method, is

1) equal to the calculated molar mass
2) more than the calculated molar mass
3) less than the calculated molar mass
4) double of the calculated molar mass

Ans : [3]

Sol : Theory

When an ionic compound like $BaCl_2$ dissolves in water it ionises (ie) the number of particles increases.

$\therefore$ The observed mol wt of any ionic compound is less than the calculated (Theoritical) Mol.wt

143. The number of moles of electrons required to deposit 36 g of Al from an aqueous solution of Al $(NO_3)_3$ is (At. wt. of Al = 27)

1) 4
2) 2
3) 3
4) 1

Ans : [1]

Sol : Theory

Eq: wt of $Al = \dfrac{Atwt}{voleng} = \dfrac{27}{3} = 9$

To deposit 9g (1gm equivalent) of

Aluminium = 1 F of electricity required

= 96500 colomb electricity required

= 1 mole of electrons required

$\therefore$ To deposit 36g Al= 4 moles of electrons required.

144. The emf (in V) of a Daniel cell containing 0.1 M $ZnSO_4$ and 0.01 M $CuSO_4$ Solutions at their respective electrodes is $(E^0_{Cu^{2+}|Cu} = 0.34V; E^0_{Zn^{2+}|Zn} = -0.76V)$

1) 1.10
2) 1.16
3) 1.13
4) 1.07

Ans : [4]

Sol : Numerical

$$E_{cell} = E^0_{cell} - \frac{0.06}{n}\log\frac{(products)}{(\text{Re}\,action)}$$

$$\left[E^0_{cell} = E^0_{cathode} - E^0_{anode}\right]$$

At anode

$Zn \to Zn^{2+} + 2e^-$

At cathode $Cu^{2+} + 2e^- \to Cu$

$$\underset{(0.01M)}{Cu^{2+}} + Zn \to Cu + \underset{(0.1M)}{Zn^{2+}}$$

$$E_{cell} = E^0_{cell} - \frac{0.06}{2}\log\frac{\left[Z_n^{2+}\right]}{\left[Cu^{2+}\right]}$$

$$E_{cell} = \left[0.34 - (-0.76)\right] - \frac{0.06}{2}\log\frac{10^{-1}}{10^{-2}}$$

$$= 1.1 - 0.03\log 10$$

$$= 1.1 - 0.03$$

$$= 1.07V$$

145. Which one of the following elements, when present as an impurity in silicon makes it a p-type semiconductor ?

1) As 2) P 3) In 4) Sb

Ans : [3]

Sol : P-type semi conductor is prepared by doping IV A group element (Si) with IIIA group like B, Ga, In,etc

146. Which one of the following statements is correct for the reaction

$$\underset{(aq)}{CH_3COOC_2H_5} + \underset{(aq)}{NaOH} \rightarrow \underset{(aq)}{CH_3COONa} + \underset{(aq)}{C_2H_5OH}$$

1) Order is two but molecularity is one
2) Order is one but molecularity is two
3) Order is one but molecularity is one
4) Order is two but molecularity is two

Ans : [4]

Sol : (Theory) Soapnification of ester is a second order reaction and molecularity of the reaction is two.

$$Rate = k\left(ester\right)^1\left(NaOH\right)^1$$

147. The catalyst and promoter respectively used in the Haber's process of industrial synthesis of ammonia are

1) Mo, V_2O_5 2) V_2O_5, Fe 3) Fe, Mo 4) Mo, Fe

Ans : [3]

Sol : Theory

$$N_2 + 3H_2 \xrightleftharpoons[\;]{\text{Fe} - \text{Catalyst}, \; M_o \text{ - promotor}} 2NH_3 \;;\; \Delta H = -22.4 kcl$$

148. Which one of the following statements is NOT correct ?

1) The pH of 1.0 x 10^{-8} M HCl is less than 7.
2) The ionic product of water at 25^0C is 1.0 x 10^{-14} mol^2 L^{-2}
3) Cl^- is a Lewis acid.
4) Bronsted - Lowry theory cannot explain the acidic character of $AlCl_3$.

Ans : [3]

Sol : Theory

Cl^- is a Lewis base and not a Lewis acid

149. The molar heat capacity (C_p) of water at constant pressure is 75 J.K^{-1}. mol^{-1}. The increase in temperature (in K) of 100 g of water when 1 K.J. of heat is supplied to it is

1) 2.4 2) 0.24 3) 1.3 4) 0.13

Ans : [1]

Sol : Numerical

Molar heat capacity (C_p) is for 1 mole to rise the temperature by 1^0C

$\therefore$ For 18g water = 75J.k^{-1}.$mole^{-1}$

$$\Delta H = nC_p.\Delta t$$
$$1000 = \frac{100}{18}.75 \times t$$
$$t = 2.4$$

150. Gelly is a colloidal solution of

1) Solid in liquid 2) Liquid in solid 3) Liquid in liquid 4) Solid in solid

Ans : [2]

Sol : Theory

Gelly is a coloidal solution of liquid in solid

151. The product (s) formed when H_2O_2 reacts with disodium hydrogen phosphate is

1) P_2O_5, Na_3PO_4 2) Na_2HPO_4, H_2O_2 3) NaH_2PO_4, H_2O 4) Na_2HPO_4, H_2O

Ans : [2]

Sol : Theory

H_2O_2 can form addition compound with Disodium hyderogen phosphate $Na_2HPO_4.Na_2HPO_4.H_2O_2$

152. Which of the following is NOT correct ?

1) LiOH is a weaker base than NaOH 2) Salts of Be undergo hydrolysis

3) $Ca(HCO_3)_2$ is soluble in water

4) Hydrolysis of beryllium carbide gives acetylene

Ans : [4]

Sol : Theory

Hydrolysis of Beryllium carbide given methane

153. What is Z in the following reactions ? $BCl_3 + H_2 \xrightarrow[450^0C]{Cu\text{-}Al} X + HCl$ $X \xrightarrow{methylation} Z$

1) $(CH_3)BH_2$ 2) $(CH_3)_4B_2H_2$ 3) $(CH_3)_3B_2H_3$ 4) $(CH_3)_6B_2$

Ans : [2]

$$BCl_3 + H_2 \xrightarrow[450^0C]{Cu-Al} \underset{(X)}{B_2H_6} + HCl$$

Sol : $$\underset{(X)}{B_2H_6} + 4CH_3Cl \xrightarrow{methylatin} \underset{(Z)}{(CH_3)_4B_2H_2}$$

154. Which one of the following elements reduces NaOH to Na ?

1) Si 2) Pb 3) C 4) Sn

Ans : [3]

Sol : Theory

$$6NaOH + 2C \xrightarrow{\Delta} 2Na + 2Na_2CO_3 + 3H_2 \uparrow$$

Here carbon act as raducing agent

155. Which one of the following is used in the preparation of cellulose nitrate ?

1) KNO_3 2) HNO_3 3) KNO_2 4) HNO_2

Ans : [2]

Sol : Theory

One of the use of HNO_3 is used in the preparation of cellulosenitrate

156. The oxoacid of sulphur which contains two sulphur atoms in different oxidation states is

1) Pyrosulphurous acid
2) Hyposulphurous acid
3) Pyrosulphuric acid
4) Persulphuric acid

Ans : [1]

Sol : Theory

157. Bond energy of Cl_2, Br_2 and I_2 follow the order

1) $Cl_2 > Br_2 > I_2$ 2) $Br_2 > Cl_2 > I_2$ 3) $I_2 > Br_2 > Cl_2$ 4) $I_2 > Cl_2 > Br_2$

Ans : [1]

Sol : Theory

Bond energy order is $Cl_2 > Br_2 > I_2$

158. Assertion (A) : The boiling points of noble gases increases from He to Xe.

Reason (R) : The interatomic van der Waals attractive forces increases from He to Xe.

The correct answer is

1) Both (A) and (R) are true, and (R) is the correct explanation of (A)
2) Both (A) and (R) are true, and (R) is not the correct explanation of (A)
3) (A) is true but (R) is not true
4))(A) is not true but (R) is true

Ans : [1]

Sol : Theory

From He to Xe the Atomic size increases. Therefore the Vanderwalls force of attraction increases . As a result the boiling point increases from He to Xe.

159. A coordinate complex contains Co^{3+}, Cl^- and NH_3. When dissolved in water, one mole of this complex gave a total of 3 moles of ions. The complex is

1) $\left[Co\left(NH_3\right)_6\right]Cl_3$
2) $\left[Co\left(NH_3\right)_5 Cl\right]Cl_2$
3) $\left[Co\left(NH_3\right)_4 Cl_2\right]Cl$
4) $\left[Co\left(NH_3\right)_3 Cl_3\right]$

Ans : [2]

Sol : Theory

$$\left[Co\left(NH_3\right)_5 Cl\right]Cl_2 \rightarrow \underset{one\ mole}{\left[Co\left(NH_3\right)_5 Cl\right]^{2+}} + \underset{2-moles}{2Cl^-}$$

$\therefore$ Totally 3-moles of ions are produced.

160. Ni anode is used in the electrolytic extraction of

1) Al
2) Mg
3) Na by Down's process
4) Na by Castner's process

Ans : [4]

Sol : Theory

In castners process of the extraction of sodium, Ni act as anode